Divergence-free shock-capturing schemes for magnetohydrodynamics

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The magnetohydrodynamic (MHD) simulation is an indispensable tool to study various macroscopic dynamics in space-terrestrial plasma environments. A common strategy to build an MHD simulation code for the space plasma would be based on a shock-capturing scheme. The shock-capturing scheme calculates an intermediate state between computational cells through an exact or approximate solution of the Riemann problem, in order to update the hyperbolic conservation laws in a divergence form. Numerous studies have been devoted to develop robust and accurate Riemann solvers for MHD equations, and currently, the HLLD approximate Riemann solver by Miyoshi and Kusano (2005) becomes a de fact standard for MHD simulations of space and astrophysical plasmas.

Many Riemann solvers are designed based on a one-dimensional problem, and they can be extended to multi-dimension by a dimension-by-dimension fashion. However, MHD shock-capturing schemes do not necessary preserve the divergence-free condition of the magnetic field, and its violation may lead to an unphysical solution. Staggered grid spacing for the magnetic field is one of the method to perverse the divergence-free condition, in which the induction equation is discretized to be consistent with the curl form (Evans and Hawley 1988). Developing a proper method to combine the shock-capturing scheme with the divergence-free scheme has been a major concern for computational MHD.

In this paper, we propose two simple methods for the divergence-free shock capturing scheme. The scheme adopts the HLLD Riemann solver and the staggered grid spacing as a building block. The critical issue is the calculation of the electric field (numerical flux of the magnetic field) at the cell corner that should be consistent with the solution of the Riemann problem. The first method is a successive one-dimensional reconstruction of the numerical flux from a cell center to a corner. The second method is a recovery of a proper amount of numerical diffusion in the familiar Flux-CT scheme by Balsara and Spicer (1999). Furthermore, we incorporate a multi-dimensional upwinding in order to obtain a less oscillatory solution in advection-dominated problems. The two methods are consistent with a one-dimensional shock-capturing scheme for one-dimensional problems. Various benchmark tests and physical problems are demonstrated to measure the capability of the proposed methods.