地中の電流源による 2 次元静電場の海洋と山脈を考慮した解析的解法

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Analytic 2-D electrostatic fields by underground electric current sources considering seas and mountains

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A certain sort of analysis of GPS-TEC suggests appearance of ionospheric disturbances prior to huge earthquakes. Especially, when considering the ionospheric perturbation prior to huge earthquakes in the subduction zone of oceanic plates, the mountains along island arcs are expected to efficiently propagate the electric field from the electric current source generated in the subduction zone to the ionosphere through the atmosphere. These expectations, however, lack considering the attenuation of the electric field along the seas which are more conducting by orders of magnitude than the solid earth. The present study shows the procedure to analytically evaluate the electric field which appears in both the ground and the air considering the existence of both the seas and mountains due to a certain electric current source in the ground, no matter what the generating mechanism is, based on assuming the simplest structure of the electrical conductivity.

The structure is assumed to be 2D of which the strike is along trench axis. The ground and the air which are assumed to be isotropic and homogeneous are regarded to be a conductor and a dielectric, respectively. The existence of the ionosphere is ignored in the present evaluation for simplicity. The seas which are parts of the boundary between the ground and the air are approximated as perfect conductors with infinitesimally small thicknesses. The shape of the mountains surrounded by the seas are expressed so that its boundary to the air on the section perpendicular to the strike coincides with two legs of an isosceles triangle of which the height of the base coincides to the sea level. The resultant boundary conditions for the electrostatic potential are: zero along the seas for both the ground and the air, and zero gradient in the ground perpendicular to the boundary between the air and the solid earth due to the disappearance of the electric current density perpendicular to the boundary.

When solving 2D electrostatic potential, expressing the section perpendicular to the strike with a complex plane, and expanding the electrostatic potential to a complex potential, its imaginary part can express the electric field lines. By obtaining the electrostatic potential caused by the electric current source and satisfying the boundary condition, the stream function expressing the electric field lines can be obtained with the electrostatic potential and the Cauchy-Riemann equation.

At the limit of the flat mountains with seas, the electrostatic potential can be obtained with the Airfoil equation (Tricomi, 1985) which is an inhomogeneous Fredholm integral equation of the first kind with the Cauchy kernel, satisfied by the tangential electric field at the boundary between the solid earth and the air. The mathematical expression of the complex potential in both the ground and the air can be obtained with the electrostatic potential on the boundary using Chebyshev polynomial expansion of the first and second kinds (Tricomi, 1985).

At the limit that the height of the mountains is non-zero and that the seas are neglected, the complex potential can be obtained with a conformal mapping of the potential from another complex plane on which the height is zero and the boundary is flat. The transformation formula is given as a Schwarz-Christoffel transformation which transforms both a part of the flat boundary to the legs of the isosceles triangle, and the rest of the flat boundary to flat boundaries, expressed with the Gauss hypergeometric function.

By the combination of the derivations of the complex potentials, the potential can be obtained considering both the seas and the mountains. The characteristics of the electrostatic potential and the electric field lines are shown in detail for two cases that the electric current source is located along inland faults and subduction zones.

巨大地震に先行する電離圏の異常の発生が GPS-TEC の解析から指摘されている。特に海溝型巨大地震に先行する電離圏の異常を考察する場合には、島弧に沿った山脈が、沈み込み帯で発生すると仮定する電流源による電場を大気中へと効率的に伝える経路となることが期待されている。しかしこのような想定においては、固体地球より良導的な海洋の存在が海底下で水平電場を減衰させる影響が無視されている。本研究は電流源の発生過程によらず、地中に何らかの電流源を想定した場合に、海洋と山脈を考慮して地表と大気中に現れうる静電場を、最も簡単な電気伝導度構造を仮定して解析的に見積もる手法を示す。

構造は海溝軸に平行な走向を持つ 2 次元構造を仮定する。大地と大気は等方均質なそれぞれ導電体と誘電体とみなす。電離圏の存在は無視する。大地と大気の境界のうち、海洋は微小厚さの完全導体と近似し、島弧を模した山脈の両側に分布するものとする。山脈の形状は、走向に直交する断面が海面高度に一致した底辺を持つ二等辺三角形で現されるものとする。このとき静電ポテンシャルの境界条件は、境界面のうち海面では大地側と大気側で共通に無限遠における値に等しくゼロ、固体地球と大気との境界では大地側では地表に直交する電流密度の消失から法線方向の勾配がゼロとなる。

2 次元の静電場を解く際に、構造の走向に直交した 2 次元断面を複素平面で表現し、静電ポテンシャルに代わって複素ポテンシャルを考察すれば、その虚部は電気力線を表現する。電流源によって生じる、境界条件を満たした静電ポテンシャルをまず解けば、電気力線を表現する流れ関数は静電ポテンシャルと Cauchy-Riemann の方程式とから導かれる。

まず山脈の標高をゼロとした極限では、平面大地と海洋とを考慮した境界条件に対して静電ポテンシャルは、地表面における接線電場が満たす、Cauchy 核をもつ非斉次第一種フレドホルム積分方程式、Airfoil 方程式 (Tricomi, 1985) から得られる。Tricomi(1985) が示す、第一種及び第二種 Chebyshev 多項式を用いた多項式展開で表現される境界面の静電ポテンシャルから地中及び大気中の複素ポテンシャルの数学的表現が得られる。

一方、山脈の標高が有限で海洋を無視した極限では、複素ポテンシャルの解は山脈の標高をゼロとした場合の解の等角写像によって得られる。写像を表現する変換式は、山脈の形状を模した境界面に平坦面を写像する Schwarz-Christoffel 変換から得られ、仮定からはガウスの超幾何関数を用いて表現される。

以上の解法の組み合わせにより、海洋と山脈の両方を考慮した場合の複素ポテンシャルを求めることが出来る。静電ポテンシャル及び電気力線の、電流源を沈み込み帯と内陸断層のそれぞれを模した位置に配置した場合の特徴について本発表で詳述する。