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A method for obtaining steady-state solutions to the particle transport equation

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Understanding the transport of energetic charged particles is a crucial problem with numerous applications in space and astrophysical plasma physics. Classical Parker's transport equation describes convection with the background plasma flow and spatial diffusion with respect to it. It has been widely used in modeling, e.g., particle acceleration at collisionless shocks, propagation of energetic particles in the heliosphere and beyond. The diffusion-convection equation is obtained by assuming pitch-angle isotropy from the more fundamental transport equation derived by Skilling (Skilling 1975), often called the focused transport equation. It can be used in a broader range of applications in which finite pitch-angle anisotropy is crucial. On the other hand, it is, in general, difficult to solve the focused transport equation with an analytic approach due to its higher dimensionality, and one often resorts to numerical methods.
In this study, we are only concerned with the steady-state solution of the transport equation under given boundary conditions, which is often the case in practical applications. One of the most straightforward and naive approaches is to integrate the time-dependent transport equation in time for a sufficiently long time such that the numerical solution approaches the steady-state. This approach is, however, clearly not the optimum in terms of numerical efficiency. The required numerical cost increases quite substantially when the system involves multiple, vastly different time scales. The reason for this is that an explicit time-integration scheme needs to resolve the shortest time scale, whereas the steady-state should be obtained well after the longest time scale in the system is reached.
We here present a practical and efficient numerical method to obtain the steady-state solution to the transport equation. Since the equation is linear, the discretized form can, in general, be represented as a matrix equation. The steady-state solution is simply obtained by inverting matrix using routines provided by standard well-optimized numerical libraries. The problem thus essentially reduces to the discretization strategy. We demonstrate that accurate numerical solution can be obtained by using the pseudo-spectral method with an appropriate choice of basis functions. More specifically, we expand the solution with the Chebyshev polynomials for each dimension and represents the matrix using the pseudo-spectral approach. We will discuss the basics of the proposed method, application to the shock acceleration theory, and its possible extensions.